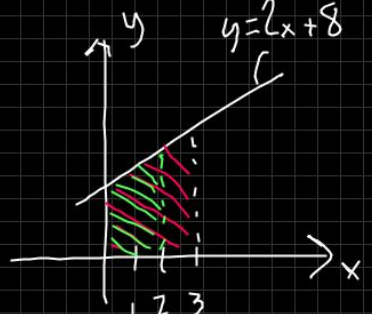
16 a) Integralen $\int_2^3 \underbrace{2(x+4)}_{f(x)}\,dx$ har värdet **13**. Visa hur du kommer fram till detta med hjälp av primitiv funktion.

$$2\int_2^3 (x+4)\,dx = 2\left[\frac{x^2}{2}+4x\right]_2^3 \quad 2(\ \ )-2(\ \ )$$

Lösning:



$f(x) = 2(x+4) = 2x+8$

$F(x) = x^2+8x+C$

$F(3) = 3^2+8\cdot 3+C = 33+C$

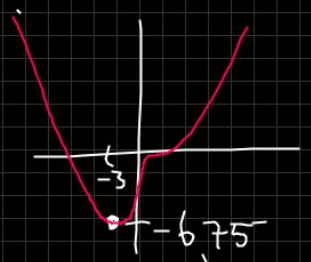
$F(2) = 2^2+8\cdot 2+C = 20+C$

Skillnaden $F(3)-F(2) =$

$(33+C)-(20+C) =$

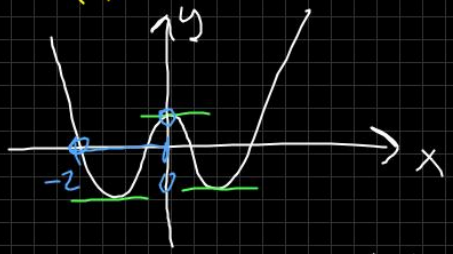
$33+C-20-C = 13$

(19) Bestäm det minsta värdet till funktionen $-2 \le x \le 0$



a) $f(x) = x^4 - 4x^2$

skiss



Lösning:

$f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x = 4x(x^2 - 2) = 4x(x+\sqrt{2})(x-\sqrt{2})$

x	$-\sqrt{2}$		0		$\sqrt{2}$	
$4x$	−	−	0	+	+	+
$(x-\sqrt{2})$	−	−	−	−	0	+
$(x+\sqrt{2})$	−	0	+	+	+	+
$f'(x)$	−	0	+	0	−	0	+
$f(x)$	↘		↗		↘		↗

$\begin{cases} x_1 = 0 \\ x_2 = -\sqrt{2} \\ x_3 = \sqrt{2} \end{cases}$

$f(-\sqrt{2}) = (-\sqrt{2})^4 - 4(-\sqrt{2})^2 = -4$

$f(\sqrt{2}) = 4 - 4 \cdot 2 = -4$

Svar: minsta värde är -4

b) $f(x) = \frac{x^4}{4} + x^3$

Lösn:

$f(x) = \frac{x^4}{4} + x^3$

$f'(x) = x^3 + 3x^2 = x^2(x+3)$ $\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = -3 \end{cases}$

x		-3		0	
x^2	+	+	+	0	+
$(x+3)$	−	0	+	+	+
$f'(x)$	−	0	+	0	+
$f(x)$	↘		↗		↗

minimum, terass

$f(-3) = \frac{(-3)^4}{4} + (-3)^3$

$= \frac{81}{4} - 27 =$

$f(-3) = -6{,}75$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2$$

$$f(x+h) = (x+h)^2$$

"Ändringskvoten"

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

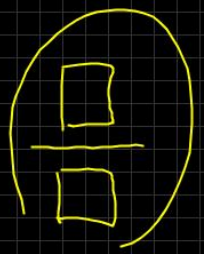
$$= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h$$

låt $h \to 0$ $\quad \lim_{h \to 0} 2x + h = 2x$

Svar: $f'(x) = 2x$

Gränsvärde $\left(\lim_{h\to 0}, \lim_{t\to 5}, \lim_{a\to\infty}\right)$

$$\lim_{h\to 0}(4-2h) = 4-2\cdot 0 = 4$$



$$\lim_{x\to\infty}\sqrt{x+x^2}-x = \lim_{x\to\infty}\frac{\left(\sqrt{x+x^2}-x\right)\left(\sqrt{x+x^2}+x\right)}{\left(\sqrt{x+x^2}+x\right)} =$$

$$\lim_{x\to\infty}\frac{x+x^2-x^2}{\sqrt{x+x^2}+x} = \lim_{x\to\infty}\frac{x}{\sqrt{x+x^2}+x}$$

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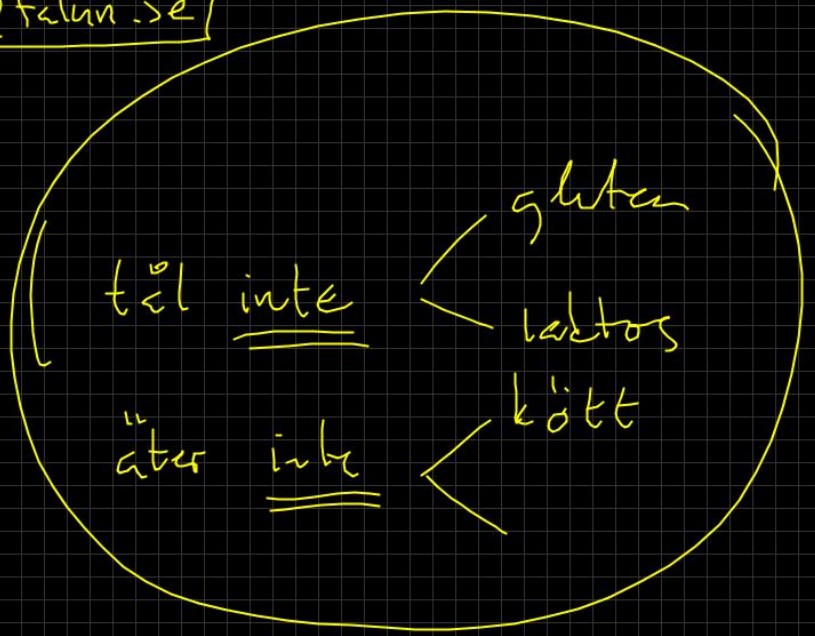
mejla mig NP

- Specialkost



- Behov av

1) lugn miljö
2) lexicon
3) längre tid