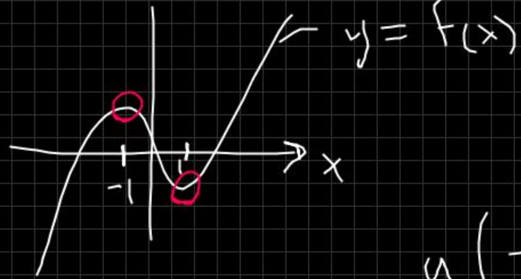
Repetition kap. 3.

(5) Bestäm koordinaterna för extrempunkterna till kurvan $y = x^3 - 3x + 5$

sid 196

"skiss"



Lösning: Tredjegradare:

$$y = x^3 - 3x + 5$$

$$y' = 3x^2 - 3 = 3(x+1)(x-1)$$

$$y' = 0 \Rightarrow \begin{cases} x_1 = -1 \\ x_2 = 1 \end{cases}$$ ins. i y

$$y(-1) = (-1)^3 - 3\cdot(-1) + 5 = 7$$

$$y(1) = 1^3 - 3\cdot 1 + 5 = 3$$

Svar: $(-1, 7)$ maximum, $(1, 3)$ minimum

10 Bestäm andraderivatan till

$$y = x^3 - 2x + \frac{1}{2x}$$

Lösning:

$$y = x^3 - 2x + \frac{1}{2} \cdot x^{-1}$$

$$y' = 3x^2 - 2 + (-1)\frac{1}{2} \cdot x^{-2}$$

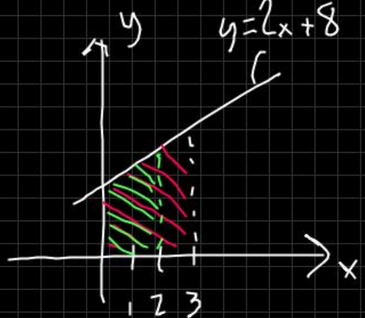
$$y'' = 6x + (-1) \cdot \frac{1}{\cancel{2}} \cdot (-\cancel{2}) \cdot x^{-3}$$

$$y'' = 6x + x^{-3}$$

Svar: $y'' = 6x + \frac{1}{x^3}$

16 a) Integralen $\int_2^3 \underbrace{2(x+4)}_{f(x)}\,dx$ har värdet 13. Visa hur du kommer fram till detta med hjälp av primitiv funktion.

Lösning:



$$f(x) = 2(x+4) = 2x+8$$

$$F(x) = x^2 + 8x + C$$

$$F(3) = 3^2 + 8\cdot 3 + C = \boxed{33} + C$$

$$F(2) = 2^2 + 8\cdot 2 + C = \boxed{20} + C$$

Skillnaden $F(3) - F(2) =$

$$(33+C) - (20+C) =$$

$$33 + \cancel{C} - 20 - \cancel{C} = 13$$

b) Bestäm det positiva talet a så att:

$$\int_2^a 2(x+4)\,dx = 493$$

Lösning:

$$\int_2^a (2x+8)\,dx = \Big[x^2+8x\Big]_2^a = (a^2+8a) - (2^2+8\cdot 2) =$$

$$a^2+8a-20 = 493$$

$$a^2+8a-513 = 0$$

Använd pq-formel.

$$a = -4 \pm \sqrt{4^2-(-513)}$$

$$a = -4 \pm \sqrt{529}$$

$$a_1 = -4+23 = 19$$

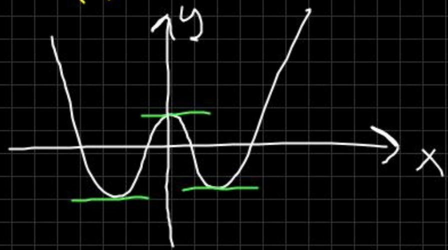
$$(a_2 = -4-23 = -27)$$

Svar $a=19$

19 Bestäm det minsta värdet till funktionen

a) $f(x) = x^4 - 4x^2$

skiss



Lösning:

$f(x) = x^4 - 4x^2$

$f'(x) = 4x^3 - 8x = 4x(x^2 - 2) = 4x(x+\sqrt{2})(x-\sqrt{2})$

$$\begin{cases} x_1 = 0 \\ x_2 = -\sqrt{2} \\ x_3 = \sqrt{2} \end{cases}$$

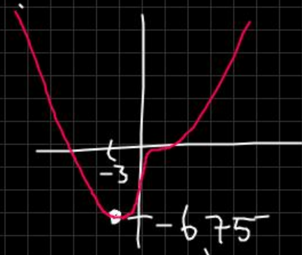
x		$-\sqrt{2}$		0		$\sqrt{2}$	
$4x$	$-$	$-$	$-$	0	$+$	$+$	$+$
$(x-\sqrt{2})$	$-$	$-$	$-$	$-$	$-$	0	$+$
$(x+\sqrt{2})$	$-$	0	$+$	$+$	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\nearrow$

$f(-\sqrt{2}) = (-\sqrt{2})^4 - 4(-\sqrt{2})^2 = -4$

$f(\sqrt{2}) = 4 - 4 \cdot 2 = -4$

Svar: minsta värde är -4

b) $f(x) = \dfrac{x^4}{4} + x^3$



Lösn:

$f(x) = \dfrac{x^4}{4} + x^3$

$f'(x) = x^3 + 3x^2 = x^2(x+3)$

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = -3 \end{cases}$$

x		-3		0	
x^2	$+$	$+$	$+$	0	$+$
$(x+3)$	$-$	0	$+$	$+$	$+$
$f'(x)$	$-$	0	$+$	0	$+$
$f(x)$	$\searrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\nearrow$
		minimum		terass	

$f(-3) = \dfrac{(-3)^4}{4} + (-3)^3 = \dfrac{81}{4} - 27 =$

$f(-3) = -6{,}75$