

YouTube-kanal

3 blue 1 brown

"X R
so"

"The essence of linear algebra"

Kolla arsnitt 1

(n 3)

Önskar redan upp, $[L_1, L_2, L_3]$

Essence of linear algebra

1/15

Vectors vs. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 9:52 Vectors, what even are they? | Essence of linear algebra, chapter 1 3Blue1Brown

Span 9:59 Linear combinations, span, and basis vectors | Essence of linear algebra,... 3Blue1Brown

Linear transformations 10:59 Linear transformations and matrices | Essence of linear algebra, chapter 3 3Blue1Brown

Matrix multiplication 10:04 Matrix multiplication as composition | Essence of linear algebra, chapter 4 3Blue1Brown

3D transformations 4:46 Three-dimensional linear transformations | Essence of linear... 3Blue1Brown

The determinant 10:03 The determinant | Essence of linear algebra, chapter 6 3Blue1Brown

(3.1)

$$X = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 4 & 3 & 2 & 1 & 5 \end{bmatrix}$$

a) Typ: Rad x Kolumn

(2 x 5)

Addition . Typen
mäste versch. liegen

b) utför additionen $X + Y$: Adderas resp. element med varandra.

$$X + Y = \begin{bmatrix} 1+5 & 2+4 & 3+3 & 4+2 & 5+1 \\ 2+4 & 3+3 & 4+2 & 5+1 & 1+5 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

(3.2)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \end{bmatrix}$$

Berechne $3A + (-2)B =$

$$\begin{bmatrix} 3 & 6 & 0 \\ 6 & 6 & 9 \end{bmatrix} + \begin{bmatrix} -2 & +2 & -4 \\ 0 & -6 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 8 & -4 \\ 6 & 0 & 19 \end{bmatrix}$$

(3.3)

Multiplication av matriser.

$$A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$$

Typ: 2×2 $\text{typ: } 3 \times 2$

Vilka av följande matrismultiplikationer
är möjliga att utföra?

 ~~AB~~ $(B \cdot A)$ $\underbrace{3 \times 2 \cdot 2 \cdot 2}$ (AA) $\underbrace{2 \times 2 \cdot 2 \times 2}$ ~~CA~~ $1 \times 3 \cdot 2 \times 2$ ~~BC~~ $3 \times 2 \cdot 1 \times 3$

$$A = r \cdot k$$

$$B = R \cdot K$$

$$A \cdot B =$$

$$\underbrace{r \cdot k \cdot R \cdot K}_{K = R}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

 $\text{typ: } 1 \times 3$ (CB) $\underbrace{1 \times 3 \cdot 3 \times 2}$

$$C \beta =$$

$1 \times 3 \cdot 3 \times 2$ 1×2

on $C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 2 \\ 1 \cdot 1 + 0 \cdot 2 + 1 \cdot 0 \end{bmatrix}$$

$$\beta = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & 0 \end{bmatrix}$$

$$C \beta = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

(3.4)

Bestimmen transponiert A^T L-L

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 0 & -7 \\ -7 & 6 \\ 6 & 2 \\ 0 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 0 & -7 & 6 & 0 \\ 1 & 3 & -7 & 6 & 2 & 2 \end{bmatrix}$$

typ: 2×6
 typ: 6×2

Vilka/Vilket av dessa elev-system (överbestämde)

3.15

har en entydig lösning, dvs en och endast en lösning. 

fler eler. an vad som köras

$$\text{a) } \left\{ \begin{array}{l} 2x + y - z = 1 \\ x - y + 8z = 0 \\ y - z = 5 \\ x + 3z = 1 \end{array} \right.$$

$$\sim \left(\begin{array}{cccc} 2 & 1 & -1 & 1 \\ 1 & -1 & 8 & 0 \\ 0 & 1 & -1 & 5 \\ 1 & 0 & 3 & 1 \end{array} \right)$$

\sim
redkemon,

$$\sim \left(\begin{array}{cccc} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Parametrisera: $z = t$

$$t = 1 \Rightarrow \begin{cases} z = 1 \\ y = 6 \\ x = -2 \end{cases}$$

b) $\left\{ \begin{array}{l} x + y = 2 \\ x + 2y + z = 5 \\ y - z = 3 \\ x + 2z = 5 \end{array} \right.$

$$\sim \left(\begin{array}{cccc} 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 5 \\ 0 & 1 & -1 & 3 \\ 1 & 0 & 2 & 5 \end{array} \right)$$

\sim
niedklemm.

$$\sim \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)$$

säulen lösung

$$(0x + 0y + 0z \neq 1)$$

(3.19)

$$\left\{ \begin{array}{l} x_1 + x_2 - 2x_3 - x_4 = 7 \\ 2x_1 + 3x_2 - 5x_3 - 3x_4 = 17 \\ -x_1 + 2x_2 - x_3 - 2x_4 = 2 \\ 3x_1 - 4x_2 + x_3 + 4x_4 = 0 \end{array} \right.$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 1 & -2 & -1 & 7 \\ 2 & 3 & -5 & -3 & 17 \\ -1 & 2 & -1 & -2 & 2 \\ 3 & -4 & 1 & 4 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 4 \\ 0 & 1 & -1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\left\{ \begin{array}{l} x_1 = 4 + s \\ x_2 = 3 + s + t \\ x_3 = s \\ x_4 = t \end{array} \right.$$

parametric form
s, t

(3,8)

Lös> elimination

$$\begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ b \end{pmatrix}$$

2×2 2×1 2×1

Lösung: a & b sollen bestimmt werden.

$$\begin{cases} 3a + 2 \cdot 3 = 3 \\ 1 \cdot a + (-2) \cdot 3 = b \end{cases}$$

$$\sim \begin{pmatrix} 3 & 6 & 3 \\ 1 & -6 & b \end{pmatrix}$$

$$\begin{array}{l} 3a = -3 \Rightarrow a = -1 \\ a - 6 = b \Rightarrow -1 - 6 = b \Rightarrow b = -7 \end{array}$$

(3.13) $\begin{cases} x + y = 3 \\ -x - y + z = 1 \\ 4x + 4y - z = 8 \end{cases}$ ~ $\begin{pmatrix} 1 & 1 & 0 & 3 \\ -1 & -1 & 1 & 1 \\ 4 & 4 & -1 & 8 \end{pmatrix}$ ~

$$\sim \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow z = 4$$

parameter form: $x_2 = t$

$$\begin{cases} x_1 = 3 - t \\ x_2 = t \\ x_3 = 4 \end{cases}$$

"and the next
longer."